University of California, Berkeley
Physics 110B Spring 2001 Section 1 (*Strovink*)

FINAL EXAMINATION

Directions. Do all six problems, which have unequal weight. This is a closed-book closed-note exam except for three $8\frac{1}{2} \times 11$ inch sheets containing any information you wish on both sides. Calculators are not needed. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. *You must justify what you do or say.* Express your answer in terms of the quantities specified in the problem. Box or circle your answer. Remember that when you are asked for the value of a vector quantity, you must supply both the magnitude and direction.

1. (40 points)
The total power $P(t)$ radiated by an ideal electric dipole $\mathbf{p}(t)$ is given by the Larmor formula

$$P(t) = \frac{1}{4\pi\epsilon_0}\frac{2|\ddot{\mathbf{p}}(t_{\rm ret})|^2}{3c^3} ,$$

where $t_{\rm ret}$ is the retarded time.

(a) (15 points) Consider a single positive charge e located at position $(x, y, z) = (d, 0, d\cos\omega t)$, where d and ω are constants. Approximate $d \ll \lambda$, where λ is the vacuum wavelength of the emitted radiation. Working to second order in the small quantity d/λ, compute the *time-averaged* power $\langle P\rangle$ radiated by this charge.

(b) (10 points) How much time-averaged mechanical work per unit time $\langle dW/dt\rangle$ must be exerted upon this charge in order to keep it moving as specified in **(a)**?

(c) (15 points) A second *positive* charge e is added, located at position $(-d, 0, -d\cos\omega t)$. What is the new *time-averaged* power $\langle P'\rangle$ radiated by both charges? Continue to work only to second order in the small quantity d/λ.

2. (35 points)
A plane electromagnetic wave is described by

$$\mathbf{E}(z, t) = \mathrm{Re}\Big(\tilde{\mathbf{E}}\ \exp\big(i(kz - \omega t)\big)\Big) ,$$

where

$$\tilde{\mathbf{E}} = E_0\big((2 - i)\hat{\mathbf{x}} + (1 - 2i)\hat{\mathbf{y}}\big) ,$$

and E_0, k, and ω are real constants. A linear polarizer is placed in the beam, and oriented so that the largest possible fraction of the original beam's irradiance is transmitted. What is that fraction?

3. (35 points)
A plane wave $U_0 \cos(kz - \omega t)$ is incident normally on a screen. Fraunhofer conditions apply. The diffracted wave is observed from $z \to \infty$ at various angles θ with respect to the z axis.

(a) (15 points) Assume that the screen has three long parallel slits with equal spacing b and equal negligible width. Compute the irradiance ratio $I(\theta)/I(\theta = 0)$.

(b) (20 points) Instead assume that the screen has five long parallel slits with equal spacing b. The slit widths are still negligible; however, they are a function of the slit location, so that the five slit areas vary according to the ratio 1:2:3:2:1. Compute the irradiance ratio $I(\theta)/I(\theta = 0)$.

4. (20 points)
A Survivor contestant tries to signal a blimp hovering nearly overhead. It is pitch dark, and his only source of light is an infinitesimal, monochromatic, isotropic-light-emitting diode (LED). The naked LED isn't quite bright enough to be seen by his blimp-borne rescuer. Remembering Physics 110B, the contestant resolves to amplify the light signal that the rescuer perceives.

(a) (10 points) The contestant stretches a large opaque plastic sheet over a flat frame and pokes a small (couple of mm dia) circular hole in it. He carefully positions the hole directly between the LED and the blimp, separated from the LED by a couple of meters. Relative to the naked LED, is it possible that the irradiance seen by the rescuer increases? If so, by what maximum factor?

(b) (10 points) Lacking a plastic sheet, the contestant disassembles his bicycle hub to obtain a small (couple of mm dia) blackened steel ball. Using a spiderweb thread, he carefully hangs the ball directly between the LED and the blimp, separated from the LED by a couple of meters. Relative to the naked LED, is it possible that the irradiance seen by the rescuer increases? If so, by what maximum factor?

5. (35 points)
In the Drüde model for electromagnetic wave propagation in a dilute material medium, electrons (of mass m and charge $-e$) satisfy the equation of motion

$$m\ddot{x} = -\gamma m\dot{x} - kx - eE_x \ ,$$

where γ is an effective damping constant, k is an effective spring constant, and E_x is an electric field component.

Working at a particular angular frequency ω, and defining the complex electric field $\tilde{E}_x$ and complex current density $\tilde{J}_x$ through

$$E_x \equiv \mathrm{Re}\big(\tilde{E}_x \exp(-i\omega t)\big)$$
$$J_x \equiv \mathrm{Re}\big(\tilde{J}_x \exp(-i\omega t)\big) \ ,$$

one can then define the complex conductivity $\tilde{\sigma}$ through

$$\tilde{J}_x \equiv \tilde{\sigma}\tilde{E}_x \ .$$

In a medium having N electrons/m^3 that are so weakly bound that k is negligible, use the above information to derive the complex conductivity $\tilde{\sigma}$ as a function of angular frequency ω.
[*Hint*: Define $x \equiv \mathrm{Re}\big(\tilde{x}\exp(-i\omega t)\big)$.]

6. (35 points)
A point charge e travelling on the z axis has position

$$\mathbf{r}(t) = +\hat{\mathbf{z}}\beta ct \ \ (t < 0)$$
$$= -\hat{\mathbf{z}}\beta ct \ \ (t > 0) \ ,$$

where β is a positive constant that is not $\ll 1$. That is, the charge reverses direction instantaneously at $t = 0$, while it is at the origin. The fields that the charge produces are viewed by an observer at $(x, 0, 0)$, where $x > 0$.

(a) (20 points) What magnetic field $\mathbf{B}$ does the observer see at $t = 0$?

(b) (15 points) At time t such that $ct = x$ (exactly!), what is the direction of the electric field $\mathbf{E}$ seen by the observer? (You need consider only the part of the total electric field which is dominant at exactly that time.) Justify your answer.